Introduction to Aircraft Control System

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Week – 04

Lecture – 20

Steady State Specification and Different Test Signals

In this lecture, we'll be studying how we can modify the steady state behavior of the system. Here you will find some standard specification which will be a function of reference input and open loop transfer function. So based on the order of the open loop transfer function, we can show suitably the reference input and we can tackle the steady state behavior of the system. Then we'll have some example on aircraft system, how we can choose suitably the reference input to reduce the steady state error of the system. Then we'll conclude the lecture. In this lecture, we'll be discussing how we can comment on the stability of the system by looking at the system transfer function, how we can comment on it, if that system is stable or not.

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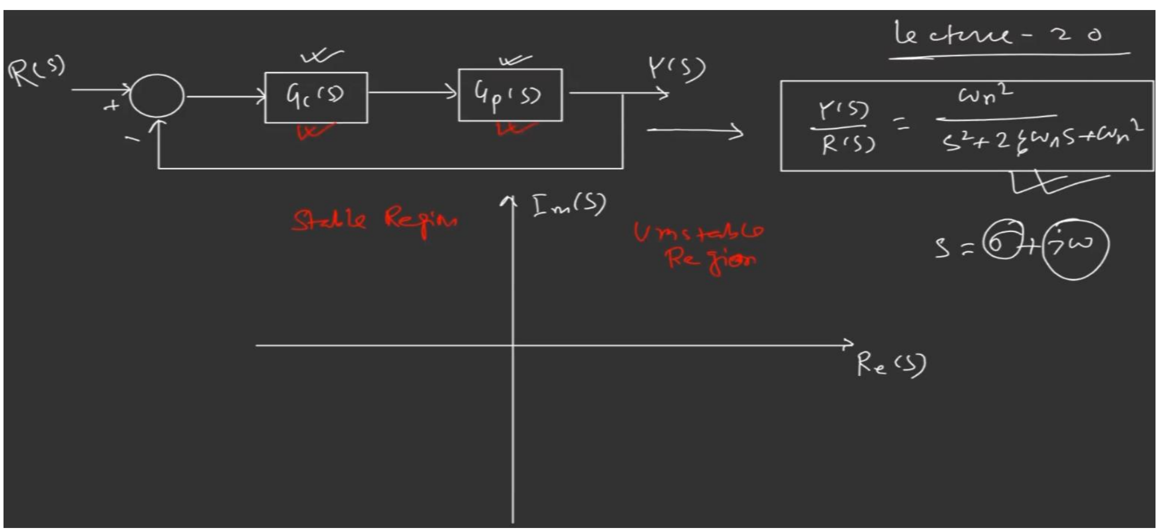

Let me remind some of the stuff we have done in the last lecture. So we had summing point and we can design the control based on the mission objective, $G_c(s)$ and this is our plant, $G_p(s)$, the output from the plant, Y(s) and step response, R(s) and the feedback loop with unity feedback. So here we can design the control algorithm for the system the system linear system and we can fulfill whether the system can track the ideal response which is

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

So this is basically the standard system, second order system and our main objective is we have to design the control algorithm here for this particular system which will follow this ideal response.

Now before you design the control algorithm here on $G_c(s)$, how we can comment on the plant, whether the plant is stable or not, whether it is stable or whether it is all the poles are on the left hand side or right hand side or any of the poles lies on left or right. So based on the location of the poles of the transfer function we can comment on the stability. Let me draw S-plane, this is the S-plane and this is real S, this is imaginary of S. Basically we can write $S = \sigma \pm j\omega$, σ is the real part and ω is the imaginary part which moves along the imaginary axis. Now the location of the poles, if the poles of the systems are lies on the right hand side, the systems assume to be unstable.

If all poles are lies on the left hand side, left hand plane then we can say that system is stable. So we can write this is the unstable region and this is the stable region. So for the stable system all poles should be lie on the left hand plane and for the unstable system if any of the poles comes out to be in the right hand side the system will be unstable. Let's assume another plot, this is $R_e(s)$, this is $I_m(s)$. If the poles lies on the right hand side on the real axis, the response will be divergent, something like this.

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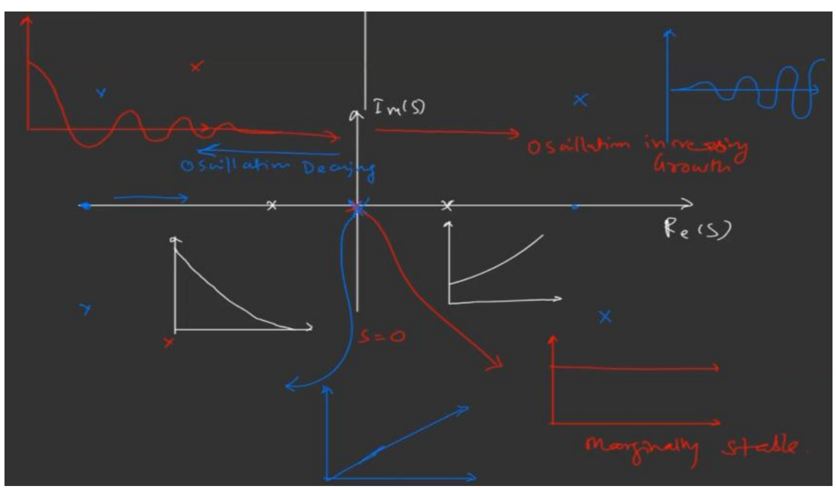

If the poles lies on the left hand side negative real axis, I can say the response would be converging to the exponentially stable system we can say. If the poles comes out to be complex conjugate in right hand plane, the system will diverging with oscillations. So it may start like this, it may diverge like this and if the poles are lies on the left hand side, left hand plane in stable region with complex conjugate poles, the response will be converging to the equilibrium point or the desired value. So it may start like this, something like this, it will go decaying oscillations. So from this we can write this part, if the pole going this direction from the imaginary axis, so we can say oscillation increasing growth if they are complex conjugate and if the poles lie goes in this direction, in this direction and the response can be oscillations decaying, oscillations decaying actually because if the poles are here, for example, so of course this negative alpha will be higher than this side.

So if the negative alpha is increasing, the speed of decaying the oscillation will be reduced very fast and if it is found to be the poles lies on the origin, if the single pole at the origin is equal to 0. For example and here the system response will be something like this, marginally stable. So this kind of response is marginally stable and if it is found to be there two repeated poles at the origin, two repeated poles at the origin, then system response will be unstable. So it is something like this. So this is how the response yields based on the location of the poles in the S-plane. So this is very, very important concept and based on the location of the poles we can design the control.

If it is found to be all the poles or one of the poles lies on the right hand side with the application of the control algorithm we can make that system stable in closed loop. So in your pole loop if it is found to be the poles are lies right and side with the application of control we can come up with the closed loop control system and you can comment on the stability of the system. So we can make the system stable if we add some poles and zeros to the system. So this is how we design the control, how we can add the poles using the application of the control. So let me draw it.

Suppose we have system, this is our plant $G_p(s)$ and this is, this plant is affected by U(s) some control and this is our control algorithm or control function we can say. So actually if it is found to be the plant is unstable we can add some pole and zeros to the system we can make the system. Closed loop system can be stable we can make. So if $G_p(s)$ is found to be unstable means if you want any of the poles lies on the right hand side system is unstable then with the control law can add poles or zeros to the system which makes unstable system unstable. So this is how we design the control algorithm.

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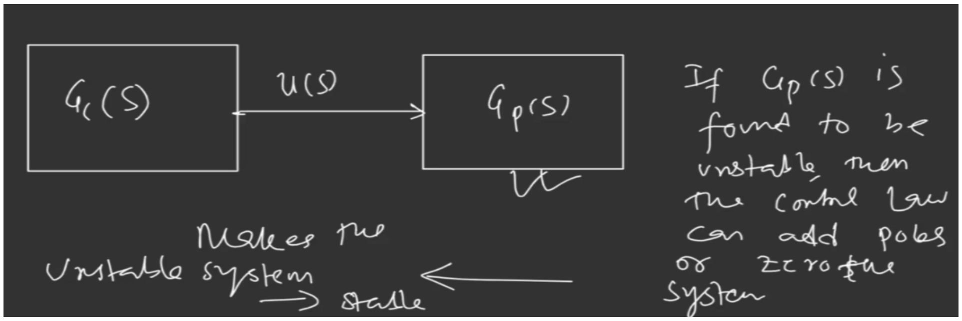

We'll be doing all these things later also how we can come up with some examples and how we can make the unstable system to stable using the control. We'll be doing the another part of transient response. So before we had this part already but I want to revise. So we have some response y(t) versus t suppose this is the signal the desired signal for example y_{final} to be tracked and my system starting from zero and going like this. So now before it goes to steady state value we call the response transient response and after it reaches to the steady state and after onwards after it reaches steady state the rest of the part we call steady state response.

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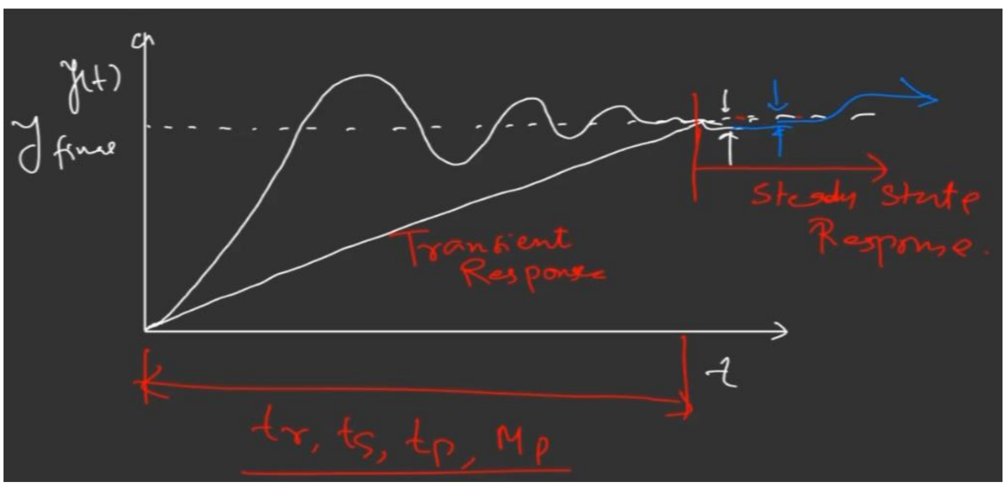

Now as of now we have discussed how we can modify this transient response using the time domain specification t_r, t_p, M_p, t_s. So this part we can modify the transient response part using the rise time, settling time, peak time and maximum overshoot. Now what about the steady state part how we can modify because in steady state the system can have some error practically there can be some error in the system this may deserve values and suppose there is some error in the system. So how can modify this steady state error using some control or any other way adding some holes and zeros how can modify it this is the part of starting now. So the title is steady state specifications.

So here we will come up with some specification as we have done for transient response to modify the steady state response in the system. Let me draw the closed loop diagram.

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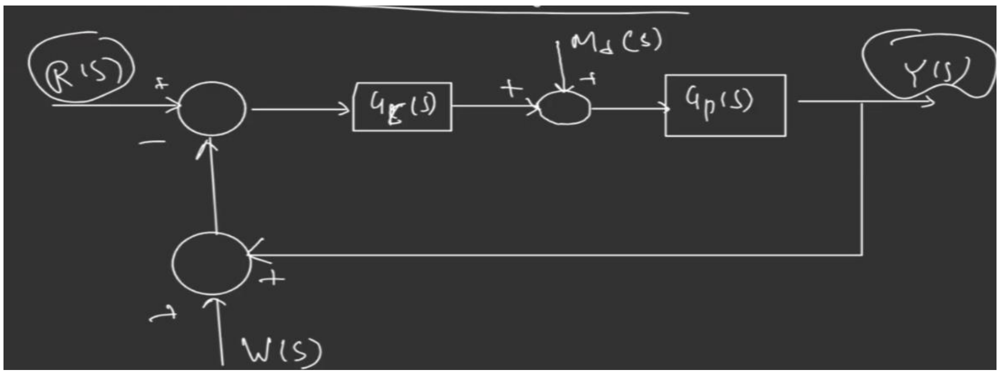

This figure is very important in control system this is the way the autopilot being designed. This is my R(s) and we have controller $G_c(s)$ and we have not used in the previous cases that disturbance but there are continuous disturbance may act in a system practically which makes the problem of a steady state error in the system. This is the output from the summing point and this is our $G_p(s)$ and this is the plant output by of S and this is our this is another summing point where noise is acting and this is unit feedback and as a control engineer we always want the Y(s) to track R(s) so always objective is as a control engineer we always want

$$e(t) = r(t) - y(t) \approx 0$$

But practically this cannot be possible because of some disturbance if you consider the aircraft system there are some always continuous disturbance acting on the system maybe wind disturbance maybe due to the engine due to some unmodeled dynamics in the system this is we there this part is difficult to achieve this errors to 0 and so due to this due to this problem, how we can make modify the steady state error in the system?

We can't make to 0 but to some extent how can modify but in practically not possible but still we can come up with some specific and which will help us to modify this steady state error so let us have the closed loop system correspondent to W(s)=0 since W(s) is the part of the frequency analysis so we'll be dealing with later now let's constraint with disturbance and reference signal the closed loop system corresponding to W(S) equal to 0 we can write Y(s) ,this is already have done how we derived it so we are not considering the W(s) part.

$$Y(s) = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}R(s) + \frac{G_p(s)M_d(s)}{1 + G_p(s)G_c(s)} \ldots \ldots Eq(1)$$

$$E(s) = R(s) - Y(s)$$

$$= R(s) - \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}R(s) - \frac{G_p(s)M_d(s)}{1 + G_p(s)G_c(s)}$$

$$= \frac{1}{1 + G_p(s)G_c(s)}R(s) - \frac{G_p(s)M_d(s)}{1 + G_p(s)G_c(s)}$$

We'll first consider this term and then we'll study on this term here if you notice R(s) is the test signal or the reference signal we have used but in practically this R(s) may not be constant or 1 by s or we can write r(t) equal to 1 but this reference signal may not be constant in some cases which we would like to track we may or may not know in advance what r(t) is going to be and even if you do know it may not have simple form that is useful for control design so therefore we'll examine now the closed loop system response to test signal which are similar to the expected reference signal r(t) so there are some other standard reference signal we can use let me start with the step input step input so where these reference signals are useful when r(t)=1 and R(s)=1/s and the response as you have done before this is our response r(t) this is t and this is basically 1 so r(t)=1 when t is greater than equal to 0 is less than 0 this is the condition for step signal right this is the appropriate when the reference signal r(t) is approximately constant for long period of time so for example we want to occasionally change the attitude from one fixed reference attitude to another.

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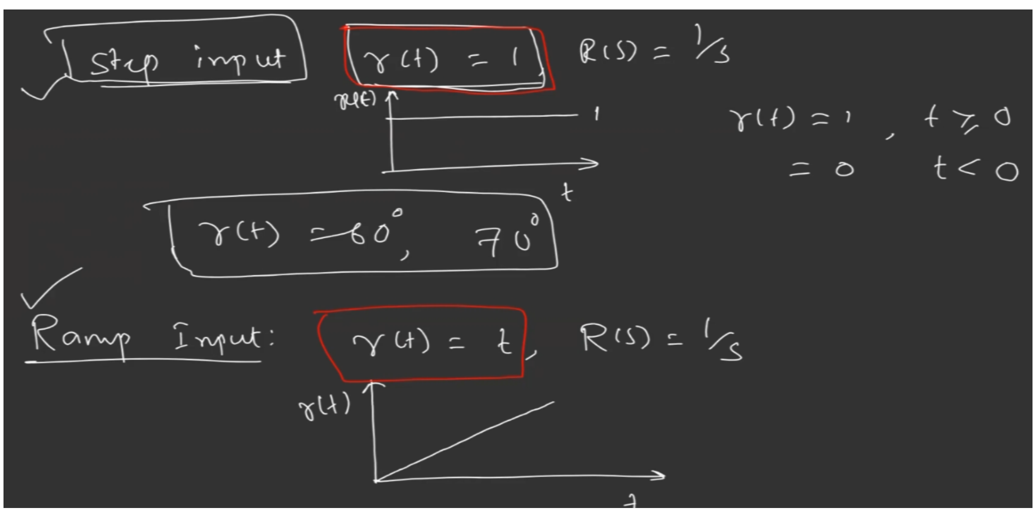

So when this kind of situation arises we can use this step signal suppose we want to give the r(t) equal to 60 degree for example and next case if you want to 70 degree some constant value we want to imply I've applied to the system as a reference signal so in that those cases we can use our step input as a reference signal now there's another signal we can use ramp input so ramp input we denote by r(t) equal to t and R(s) equal to 1 upon s squared and if you see in response of r(t) it's look like this so this is appropriate when the reference signal r(t) is slowly varying. why you are talking this type of signal will come very soon why the reason behind it because r(t) equal to 1 this is already we have done why this is the step response for this for when we are using r(t) equal to 1 so when you are using r(t) equal to t, this is basically a ramp response how the system response going to be this ramp input and so in this case if you want to when you have to use this Ramp input we want to aircraft to perform a slow attitude maneuver so when we need the slow maneuver of the orientation of the aircraft we can use r(t)=t ramp input there another reference signal we can generally use I should not say generally we don't use very often actually acceleration input $r(t) = t^2/2$ or $R(s) = 1/s^3$.

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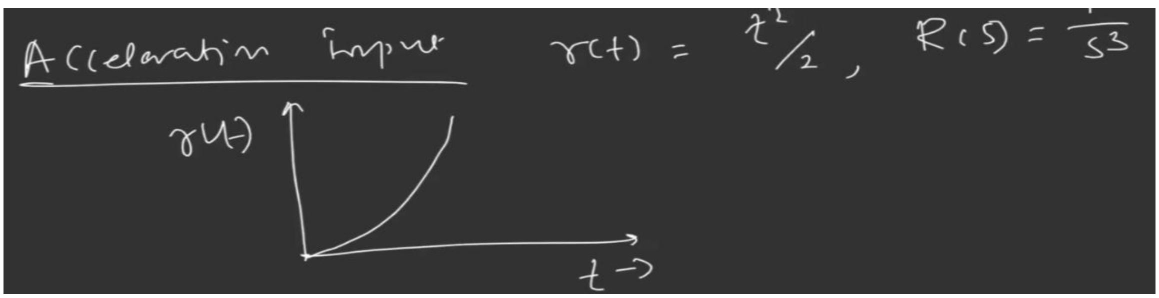

We can write something like r(t) versus t so this acceleration input signal generally not commonly used so in most of the autopilot we generally use steps input or ramp input for designing the autopilot and but step input very common and we can test the system or controller closed loop control system whether it is perfectly working or not but ramp input when we need the slowly varying aircraft that in that case we can use this r(t) quality ramp input now we'd like to discuss more on the this part how we can this let me write this equation number two so using these signals step input ramp input or acceleration input how we can modify the system response in steady state how we can modify in steady state part how we can modify this is the objective before we move to that part let me discuss on the type of the system because this is very important how we can define the type of the system based on the type of the system we can choose what kind of test signal we should refer as a reference input for the controller design.

We'll be using this first term since we are going to work on the reference signal so this part will be taken later so for the timing we'll consider if you ignore $M_d(s) = 0$ we can write

$$E(s) = \frac{1}{1 + G_p(s)G_c(s)} = \frac{1}{1 + G_0(s)} R(s)$$

Where $G_0(s)$ represents open loop transfer function. So how can define this for us in the closed loop system suppose we have this is my summing point this is the controller block $G_c(s)$and this is my plant $G_p(s)$ and this is let's assume one sensor is here which starts a function H is in the earlier cases we assume H(s) is one but if there is H the sensor transfer function is there then how we can define the open loop transfer function this is R(s) this is Y(s) now the open loop transfer function.

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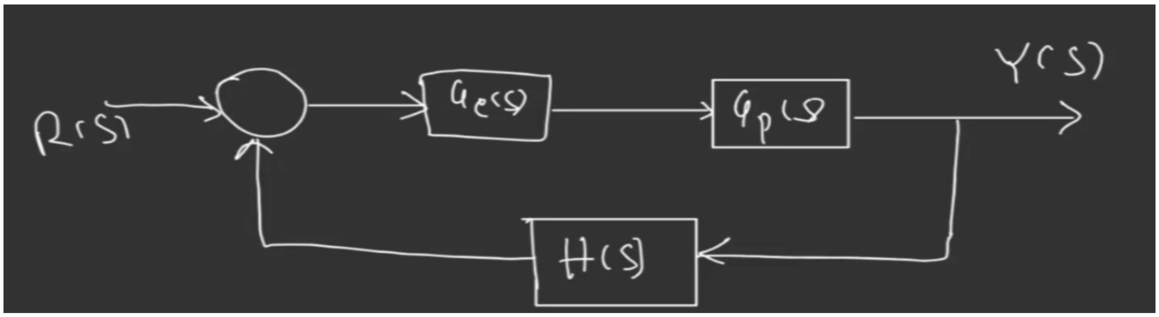

Let me define then I'll explain the open loop transfer function to be the product of product of all transfer function around the loop so in this case in this figure that open loop transfer function we can write

$$OLTF = G_0(s) = G_c(s)G_p(s)$$

For this case we have assumed the sensor transfer function is one so that's why it is like that now in general the oltf can be written as

$$OLTF = \bar{k} \frac{(s - z_1) \dots (s - z_m)}{s^N (s - p_1) \dots (s - p_n)}$$

Where z_i and p_i represents zeros and poles respectively. So this this open loop transfer function is written in terms of poles and zeros so and this is the gain which is coming from the transfer function we'll discuss later while we'll be taking an example you'll understand so here based on the this number of poles at or because this indicates s equal to zero right if it is one s is equal to zero and other poles some numerical value is there but for this part basically is equal to zero it indicates number of poles at origin actually describe or classify the type of the sensor the type of the system which type of the system so based on the number of the poles at origin we define the what type of system if there is only one s then type zero system at origin then if it is s only it means n equal to one then type one system and if it is s the zero it is type zero system if it is a square then type two system so if there is no poles for example at origin then s to their n will be one because n equal to zero then s to the n will be one so this is how we define the type of the system let's stop it here we'll come to from the next lecture how we can come up with some steady state specification which will help us to modify the steady state behavior of the system. Thank you.